

2. A curve C has equation $y = e^{4x} + x^4 + 8x + 5$
- (a) Show that the x coordinate of any turning point of C satisfies the equation

$$x^3 = -2 - e^{4x} \quad (3)$$

- (b) On the axes given on page 5, sketch, on a single diagram, the curves with equations
- (i) $y = x^3$,
- (ii) $y = -2 - e^{4x}$

On your diagram give the coordinates of the points where each curve crosses the y -axis and state the equation of any asymptotes. (4)

- (c) Explain how your diagram illustrates that the equation $x^3 = -2 - e^{4x}$ has only one root. (1)

The iteration formula

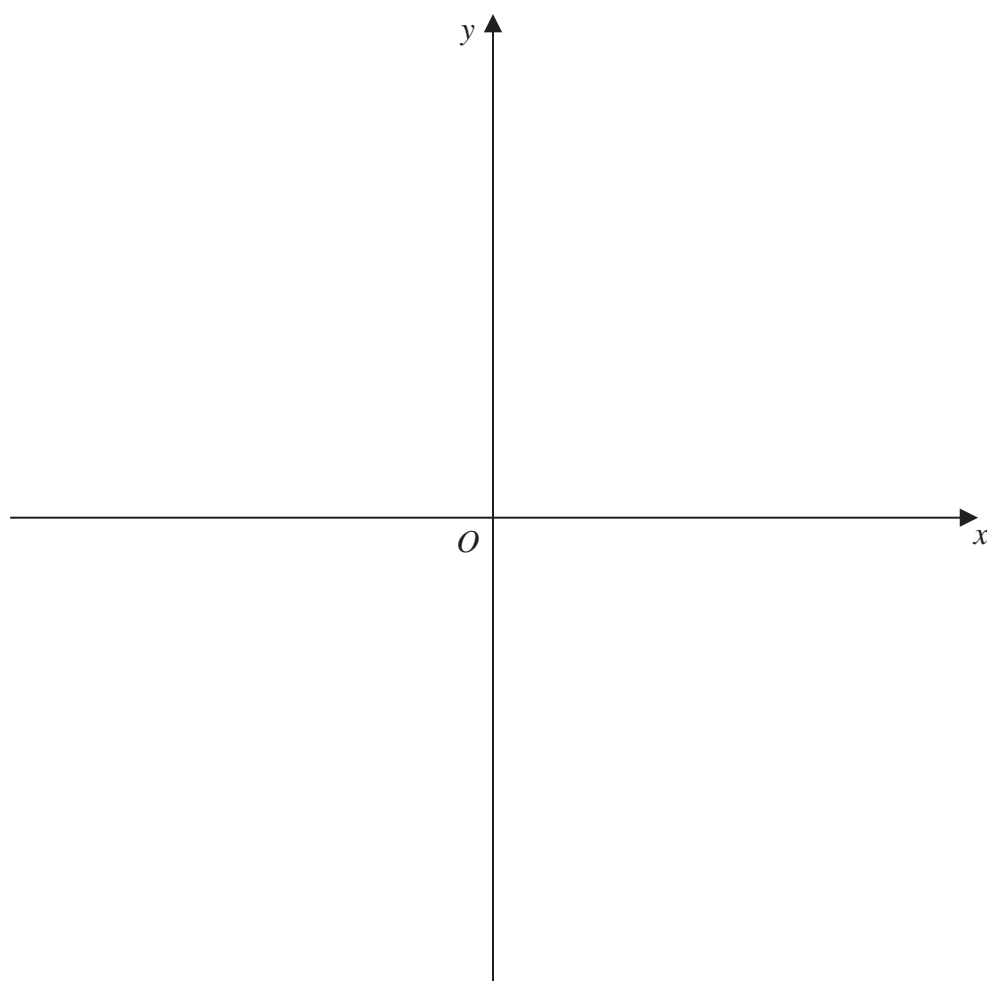
$$x_{n+1} = (-2 - e^{4x_n})^{\frac{1}{3}}, \quad x_0 = -1$$

can be used to find an approximate value for this root.

- (d) Calculate the values of x_1 and x_2 , giving your answers to 5 decimal places. (2)
- (e) Hence deduce the coordinates, to 2 decimal places, of the turning point of the curve C . (2)



Question 2 continued





3. (i) (a) Show that $2 \tan x - \cot x = 5 \operatorname{cosec} x$ may be written in the form

$$a \cos^2 x + b \cos x + c = 0$$

stating the values of the constants a , b and c .

(4)

(b) Hence solve, for $0 \leq x < 2\pi$, the equation

$$2 \tan x - \cot x = 5 \operatorname{cosec} x$$

giving your answers to 3 significant figures.

(4)

(ii) Show that

$$\tan \theta + \cot \theta \equiv \lambda \operatorname{cosec} 2\theta, \quad \theta \neq \frac{n\pi}{2}, \quad n \in \mathbb{Z}$$

stating the value of the constant λ .

(4)



4. (i) Given that

$$x = \sec^2 2y, \quad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{1}{4x\sqrt{(x-1)}} \quad (4)$$

(ii) Given that

$$y = (x^2 + x^3)\ln 2x$$

find the exact value of $\frac{dy}{dx}$ at $x = \frac{e}{2}$, giving your answer in its simplest form. (5)

(iii) Given that

$$f(x) = \frac{3\cos x}{(x+1)^{\frac{1}{3}}}, \quad x \neq -1$$

show that

$$f'(x) = \frac{g(x)}{(x+1)^{\frac{4}{3}}}, \quad x \neq -1$$

where $g(x)$ is an expression to be found. (3)

Question 4 continued

Ruled lines for writing the answer to Question 4.



5. (a) Sketch the graph with equation

$$y = |4x - 3|$$

stating the coordinates of any points where the graph cuts or meets the axes.

(2)

Find the complete set of values of x for which

- (b)

$$|4x - 3| > 2 - 2x$$

(4)

- (c)

$$|4x - 3| > \frac{3}{2} - 2x$$

(2)



Question 5 continued



6. The function f is defined by

$$f : x \rightarrow e^{2x} + k^2, \quad x \in \mathbb{R}, \quad k \text{ is a positive constant.}$$

- (a) State the range of f . (1)
- (b) Find f^{-1} and state its domain. (3)

The function g is defined by

$$g : x \rightarrow \ln(2x), \quad x > 0$$

- (c) Solve the equation $g(x) + g(x^2) + g(x^3) = 6$
giving your answer in its simplest form. (4)
- (d) Find $fg(x)$, giving your answer in its simplest form. (2)
- (e) Find, in terms of the constant k , the solution of the equation

$$fg(x) = 2k^2 \quad (2)$$



7.

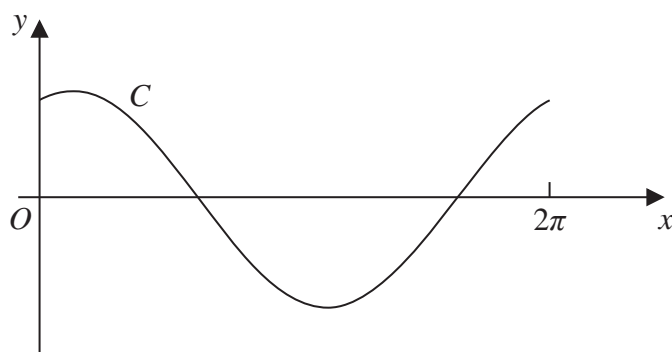


Figure 1

Figure 1 shows the curve C , with equation $y = 6 \cos x + 2.5 \sin x$ for $0 \leq x \leq 2\pi$

- (a) Express $6 \cos x + 2.5 \sin x$ in the form $R \cos(x - \alpha)$, where R and α are constants with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of α to 3 decimal places. (3)

- (b) Find the coordinates of the points on the graph where the curve C crosses the coordinate axes. (3)

A student records the number of hours of daylight each Sunday throughout the year. She starts on the last Sunday in May with a recording of 18 hours, and continues until her final recording 52 weeks later.

She models her results with the continuous function given by

$$H = 12 + 6 \cos\left(\frac{2\pi t}{52}\right) + 2.5 \sin\left(\frac{2\pi t}{52}\right), \quad 0 \leq t \leq 52$$

where H is the number of hours of daylight and t is the number of weeks since her first recording.

Use this function to find

- (c) the maximum and minimum values of H predicted by the model, (3)
- (d) the values for t when $H = 16$, giving your answers to the nearest whole number.

[You must show your working. Answers based entirely on graphical or numerical methods are not acceptable.] (6)



